

2024-09-07

(Fall-2024) 07-09-2024

Electromagnetic Part I (Final Exam)

Time: 2.00 hours

Note: This is an open notes, open book, closed friend, open mind test. On your desk you should have not any internet-enabled devices such as a computer or mobile phone.

Q1: (40 points)

- Express as a vector function the gradient (maximum directional derivative) of the following scalar fields

$$E(\rho, \varphi, z) = 15\rho^2 \cos\varphi + 200\rho \sin\varphi + 100z \cos\varphi$$

$$G(\rho, \varphi, z) = 75\rho^2 z \cos\varphi + 50\rho \sin\varphi$$

Find the magnitude of ∇E and ∇G at $P_1(2, 45^\circ, 1)$ and determine the expression for the unit vector $\mathbf{a}_{\nabla E}$ and $\mathbf{a}_{\nabla G}$. Find the angle between ∇E and ∇G at $P_1(2, 90^\circ, 1)$. (20%)

- Figure 1 shows, A spherical shell of charge possesses the constant volume charge density ρ_v between its inner and outer radii a and b . Find the E field for $r < a$, $a < r < b$ and $r > b$. (20%)

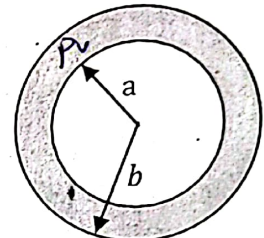


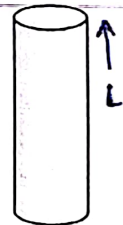
Fig.1

Q2: (30 points)

Assuming the same right circular cylindrical region of radius $\rho = a$ and length L as its shown in Fig.2. Illustrate the correctness of the divergence theorem for this region, given the electric field :

(a) $E(\rho, \varphi, z) = a_\rho \rho^3 / 4\epsilon_0 a^2$. (15%)

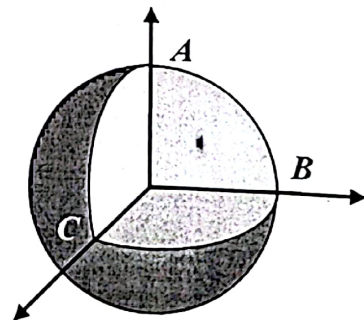
(b) $E(\rho, \varphi, z) = a_\rho a / 4\epsilon_0 \rho$ (15%)



Q3: (30 points)

A φ -directed field is defined by $G(r, \theta, \varphi) = a_\varphi 5r \sin\theta \sin\varphi$ in a region of space.

- Find **curl G** at any point. (10%)
- Evaluate the integral $\int G(r, \theta, \varphi) ds$ over the surface S of a sphere of radius $r=R$ appearing within the first octant as shown, bounded by the closed path ABC . (10%)
- Find the answer to (b) another way by use of Stokes's theorem. (10%)



-- Good Luck --